LABORATORY INSTRUCTION - RECORD PAGES

Laboratory Experiments With

Attractors

Using a Variety of Computer Programs

Geology 200 - Evolutionary Systems
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Purpose

An attractor is any state toward which a system naturally evolves. Attractors exist around us everywhere, in all forms, and we want you to begin to look for, and see, attractors for what they are when you see them, in any form that you see them. The attractors we examine here are all mathematical ones, but we explore these qualitatively, not quantitatively (i.e. you are not going to have to understand and explain how the equations work; we assume the computer is smart enough to calculate them for us.) But at the end we have a class seminar, and try to generalize the attractor concept out to the world.

Rememder, we identified 3 general types of attractors, listed below. (If you need further review go to the handout: **ATTRACTORS: STRANGE AND OTHERWISE** in the notebook.) In addition to these we briefly explore in the experiments two other types of attractors, the spiral and the circular.

- **FIXED POINT -** a steady state system. The attractor is a single point, all trajectories spiral inward toward it, e.g. a pendulum steadily losing energy to friction until there is no motion at all and the pendulum hangs vertically, held fixed by the single point which is the center of gravity.
- **LIMIT CYCLE (OR PERIODIC)** - a system which repeats itself, exactly, continuously.
- **STRANGE (CHAOTIC) -** an attractor in which the trajectory of the points circle around a region of phase space, but never exactly repeat their path. That is, they do have a predictable overall form, but the form is made up of unpredictable details. Turbulence is an example with multiple swirling eddies which go round and round the same spot, but that never repeat exactly. They may also periodically change their direction of rotation.

Notes and Comments

- When it comes time to do the formal experiments **follow the instructions below**. They are designed to systematically lead you through a series of observations.
- We begin exploring mathematical attractors where the behavior is controlled and pretty straight forward, at least as straight forward as a chaotic system can be. Toward the end, however, we move to complex systems where the behavior is elaborate enough the systems must be observed carefully to see the attractors operating.

Both the Instructions for the experiments and the spaces for recording your experimental results are contained here.

Opening the Programs

- All the programs are available in the Geology Department computer lab, Miller 232.
- Open the **Evolutionary Systems** widow in Windows; double click the icon for the program you want. A few of the programs are imbedded within one icon. Special instructions for getting to the right program are given with the program.

X-next and Attractors

By now you already have a good feeling for the X-next logistic equation, and what we ask you to do here will not surprise you. What we want you to do, however, is just look at the results of the calculations through new eyes and try to see the output in terms of attractors.

1. Experiment One - X-next Attractors at 100 Iterations

- Begin with **Iterations** of **100**, and "**r**" of **0.2** . We want you to find the ranges of "r" where X-next is in a particular attractor state.

1. Experimental Record One - X-next Attractors at 100 Iterations

Write down the ranges of "**r**" where each attractor resides.

None:	r = _________	°	r = __________	(essentially an asymptotic attenuation; see "View Output")
Point:	r = _________	°	r = __________	
Oscillating:	r = _________	°	r = __________	
Chaotic:	r = _________	°	r = __________	
Extinction:	r = _________	°	r = __________	

What do the following attractors look like on the screen. Make a sketch.

POINT: at "r" of _____	OSCILLATING: at "r" of _____	STRANGE: at "r" of _____

Are these attractors in time series, or phase space? By What criteria? Can you sketch the differences?

We did our calculations out to only 100 iterations which, as our earlier experiments with attenuation demonstrated, is not enough to really know the long term fate of some attractors.

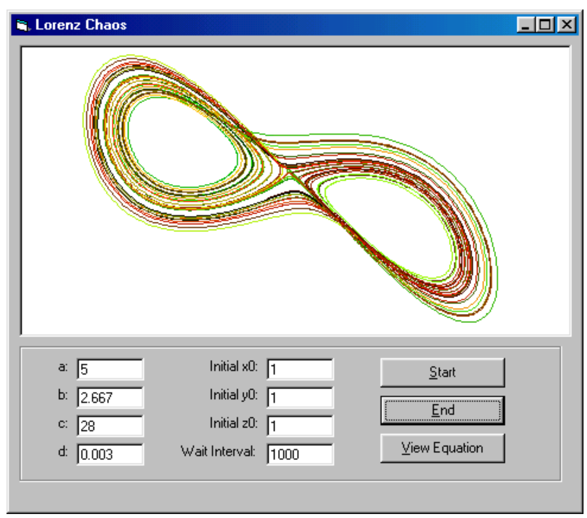
Describe the effects of observation time and attenuation on the recognition and identification of attractors. Experiment around if you need to to answer this question.

The Lorenz Attractor

Programmed by Steve J. Baedke

Exploring the Lorenz Program

- **Lorenz** is a Windows based program.
- In principle the program is not unlike the X-next equation, but has more variables, and exists in 3-dimensions rather than 2. Observe on the Lorenz screen the following:
  - Variables a, b, c, d and their default values.
  - ***Initial*** x_0, y_0 and z_0. These are the starting values on the 3-dimensional x-y-z graph (phase space).
  - ***Start:*** Every time you click Start the program starts the calculations over again at the initial values.
  - ***End:*** Stops the calculations but leaves the screen up.



- L *View Equation:* shows the Lorenz equations used here.
- L *Wait Interval:* time between calculations (depends on individual computer speeds).
- L To exit, click the **:** box in the upper right.

The Lorenz attractor was discovered in 1963 by Edward Lorenz who was trying to model the behavior of the atmosphere. It was the first chaotic (strange) attractor discovered and possesses the shape of a butterfly, hence the "butterfly effect."

The Lorenz Equation	
$x_1 = x_0 + d * a * (y_0 - x_0)$	a = 4.5
$y_1 = y_0 + d * (x0 * (c - z_0) - y_0)$	b = 2.666667
$z_1 = z_0 + d * (x_0 * y_0 - b * z_0)$	c = 28
	d = .003

Lorenz's original work had 3 equations with 3 variables, but this version has 3 equations with 4 variables (a, b, c, d); box to the right. We do not want to explore the mathematics of this system, but rely on the program to show us the outcomes so we can qualitatively evaluate what is going on.

2. Experiment Two - Observing the Lorenz Attractor

' Begin with the default conditions.

- L *Try This:* Just observe the program as it runs. Notice that it starts at one point, circles about a loop for a while, and then circles around the second loop, eventually returning to the first loop, etc..
- L Observe that the screen periodically refreshes itself, that is, clears the lines of the attractor. The program does not start over, but continues drawing where it left off. This keeps the screen from getting too messy.
- L Also, click START over and over and notice that the attractor always starts at the same initial point.

2. Experimental Record Two - Observing The Lorenz Attractor

Write down the default values. a = __________ b = __________ c = __________ d = __________	In the space below sketch the Lorenz attractor (Note that the Lorenz attractor exists in 3-dimensions, but is drawn on the screen in only 2-dimensions.)
Described what is it about the behavior of this object that makes it a strange attractor.	

Is this attractor in time series, or phase space? By What criteria?

3. Experiment Three - Varying Variable "a"

' In this experiment we are going to systematically change one of the variables and observe what happens.

3. Experimental Record Three - Varying Variable "a"

Variable "a" default is 5.0. Systematically increase "a" by values of 1 (e.g. 6, 7, 8, etc.) until you think nothing more is going to happen.	Name and describe or sketch the attractor which results.
Now systematically **decrease Variable "a"** by values of 1.0 from its default of 5.0	Name and describe or sketch the attractor which results.

4. Experiment Four - Variables b, c, and d

' Now, does increasing or decreasing the other 3 variables produce the same effects?

4. Experimental Record Four - Variables b, c. and d

For each variable below, describe the effects. Choose what ever values you deem best. Feel free to experiment and play around to get a feel for the system before making your decisions.	
Variable "b" Increasing	Decreasing

Variable "c" Increasing	Decreasing
Variable "d" Increasing	Decreasing

5. Experiment Five - Really Getting Pushy.

' If you did not do it in your earlier experiments we want you to now take some of these variables and push them really hard to see what happens.

5. Experimental Record Five - Really Getting Pushy

Change each variable below as suggested, and describe the effects. Feel free to experiment and play around.
Variable "a" Increase the "a" variable by 10's beginning at 10 until a change occurs. What was the change? Can you describe it in terms of attractors?
Now keep pushing "a" higher and higher until another change occurs. Start off gradually, but increase your increments (i.e. jumps of 50, 100, etc.) as you need to get results. What was the change? Can you describe it in terms of attractors?
Variable "d" Decrease and increase the "d" variable by one decimal place, e.g. .003 to .03, or .003 to .0003 until changes occur. Describe the changes in terms of attractors.

Do you think variables "b" and "c" will behave the same way? Why or why not?

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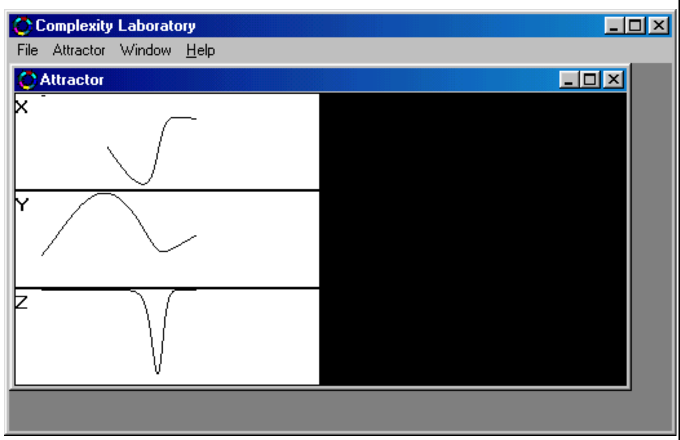
The Complexity Lab's Julian Attractor

Programmed by William H. Roetzheim [1]

Opening the Complexity Lab's New Attractor Program

- When the Windows Program Screen comes open:
  - Double Click on the **Complexity Lab** icon to bring that window up front.
  - Click FILE: NEW ATTRACTOR to bring up the program.

Exploring the New Attractor Program



- Go to ATTRACTOR: SETUP and peruse the screen:
  - Lower right shows two attractors to choose from, **Julian** and **Lorenz**. We only use the default Julian attractor here, but you may play with the Lorenz if you want (see Help file).
  - ***Initial*** x_0, y_1, and z_0. Note these two attractors have the standard 3-dimensional x, y, and z axes.
  - ***Coefficients:*** There are only 3 variables (coefficients) here, A, B, and C (instead of the four we used with the last program).
  - ***Speed:*** "0" is the fastest; the higher the number the slower it is; to really low it down try changing by 100's, or just jump to 1000.
  - ***Points*** to draw is essentially how long the program calculates and plots points.. Just leave them at the defaults.

[1] This program came with the book Enter the Complexity Lab: Where Chaos Meets Complexity by William H. Roetzheim, Sams Publishing, 1994, 208 pages (with computer disc). The book is now out of print..

‘ Go to HELP: TOPIC SEARCH which brings up some suggestions for making attractors. There are two things to note here.
- Î Descriptions of various kinds of attractors. Roetzheim is not using exactly the same types we are using, but they are close enough you can figure them out.
- Ï Suggested values for either the Julian or the Lorenz attractor to illustrate what that attractor looks like when drawn on the phase space.

6. Experiment Six - The Julian Attractor

‘ We have two very specific goals here:
- Î To learn to recognize a Julian Attractor.
- Ï To see attractors plotted in a different format, and learn to recognize them.

6. EXPERIMENTAL RECORD SIX - THE JULIAN ATTRACTOR

Write down the default values. a = __________ b = __________ c = __________	In the space below sketch the Julian attractor (Note that the Julian attractor exists in 3-dimensions, but is drawn on the screen in only 2-dimensions.)
Go to ATTRACTOR: RUN and describe what is it about the behavior of this object that makes it a strange attractor?	

7. Experiment Seven - The Circular vs. Julian Attractors

‘ Go To HELP: TOPIC SEARCH and read about the Circular Attractor.

‘ Set the **Initial Coordinates** for the Julian equation to, **1, 1, 1**; the **Coefficients** to **0.2, 0.2,** and **2.0**

‘ Go To: ATTRACTOR: RUN and observe the outcome. You may want to do it several times, or set the speed to slow it down.

7. Experimental Record Seven - The Circular vs. Julian Attractor

Write down the values for a circular attractor. x = ____ a = ____ y = ____ b = ____ z = ____ c = ____	Sketch the attractor.

Run the attractor again, but this time focus on the X, Y, and Z boxes on the left. They are plotting out the calculated values for each individual 3-D axis.

Run this as many time as you need to simultaneously observe all three graphs, X, Y, and Z (now there is a trick!), OR slow the processor speed down.

L Notice that each axis plots out a wave form. The circle you see to the right is the sum total of all 3 axes.

Your task here is to observe all three axes and describe the relationships among them. How well synchronized are they? Sketch the trajectories for X, Y and Z.

Value X

Value Y

Value Z

Now, compare the circular attractor with the Julian attractor.

Set both attractors up on the same screen to observe simultaneously by doing the following:

Î Expand the Complexity Laboratory screen to full size (little box in upper right hand corner.)

Ï Go to File: New Attractor and click it to open a second attractor screen. Drag and size this new screen so it does not overlap the first.

Ð Notice if you click the top bar of the attractor window which is gray/green, it turns red making it the active window.

Ñ You can control each window separately, and run them more or less simultaneously.

Now set up the default values for the Julian attractor (**Initial x=0, y=1, z=0**, and **a=0.2, b=0.2, c=5.7**) and compare the circular and Julian attractors. How are they similar and different? Sketch the differences between them.	
X - Circular	X - Julian
Y	Y
Z	Z

8. Experiment Eight - The Spiral Attractor (Just for Fun)

‘ Go To ATTRACTOR: SETUP and for the Julian attractor set the Initial values at 0, 1, 0 and the Coefficients at 0, 0, 0.

8. Experimental Record Eight - The Spiral Attractor

Observe what is happening at the X, Y, and Z graphs as the spiral progresses out. What is happening, and why?

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Galaxy Attractors [2]

The Galaxy Program

"Galaxy is a computer program which simulates the motions of stars under the influence of gravity. You may create a field of stars to begin with, and then watch how the stars move as they are accelerated by their mutual gravitational attractions. You may vary the number of stars, their initial velocities around a central position, and the magnitude of the Gravity Constant, which determines the attractive force between every pair of stars. You may witness how the attractive forces accelerate individual stars and send them careening off into new directions. You may also witness how large groups of stars develop into interesting patterns of distribution over time. You can see in a few minutes what would require millions of years for real stars." *(From the HELP page in the program.)*

As with many of the programs we explore, there are lots of things that can be done with this program. However, our main interest is a demonstration of attractors. Unlike the mathematical attractors above, however, Galaxy, more closely models a real system and we begin to deal with some of the messiness that comes from trying to "read" real systems.

The equations Newton wrote to describe the behavior of objects in a gravitational field are very straight forward, deterministic equations. They are non-linear, however. The nonlinear nature can be seen in the equation for the force of gravity.

$$F_g = \frac{m_1 \times m_2}{d^2}$$

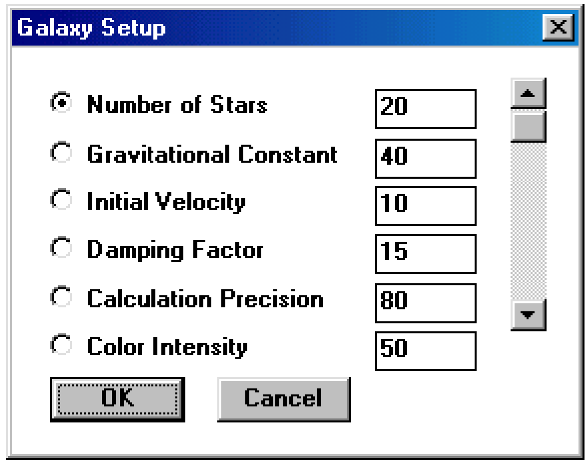
The force of gravity varies as the inverse square of the distance between the objects (an exponential variable), and so the force among three or more objects varies in very complex ways as the pull on one object by the other two objects varies exponentially, and independently, as their distances change. The system is so sensitive to these exponentially varying pulls of gravity that we cannot predict what will happen from moment to moment, let alone into the future. The general principle is that any non-linear system exhibits this sensitive dependence, and the behaviors you observe in the Galaxy program are similar to the behavior of other systems. But, as seems to be true with complex systems, the more complex the system becomes (here the more stars there are) the more regular and recognizable its behavior becomes. In this respect its behavior is like a fountain, uncountable trillions of water molecules that show a recognizable, but never exactly the same, pattern.

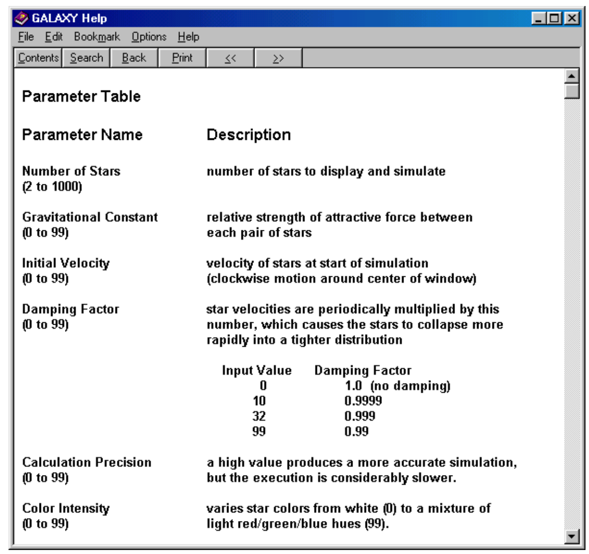
But as with most things, being told about it, or seeing an equation, is not the same as experiencing it. Hence, this experiment with Galaxy, and the computational viewpoint.

[2] Galaxy, along with other A-Life programs came with the book Enter the Complexity Lab: Where Chaos Meets Complexity by William H. Roetzheim, Sams Publishing, 1994, 208 pages (with computer disc). The book is now out of print.

Exploring Galaxy:

- **Galaxy** is a Windows based program. Observe the tool bar at the top. Move the cursor across "File" and "Setup", to see what is there.
  - SETUP: You can choose a galaxy system with anywhere from 2 to 1000 stars. Try this: in Setup click for the 20 star default. (The other variables such as "gravitational constant" and "initial velocity" are set at the optimum. See Help if you want to know how these work.)
  - INIT "initializes" a new simulation with a new star field; i.e. you can start over anytime with a new, random array of stars of the number you chose initially. Click it several times to see new star fields of 20 stars appear.
  - RUN turns the same simulation on and off. Click it several times.
  - FILE: Open contains some prepared files with established parameters.
  - HELP: PARAMETER NAMES: the image capture above. Shows the control knobs for the program.





- Observe that although the screen is two dimensional, the star field is in fact three dimensional and so stars can pass in front of and behind each other on the screen.

9. Run Nine - A Complex System

- ***Try This***: Go to FILE: OPEN: ***atom.ini***, then to SETUP = 50 stars, and then RUN.
- Let the simulation go for a while.
  - Try several INIT: RUN's.

What you are observing is a complex system. Clearly it has a lot of energy. Fifty stars orbiting around each other. And it is an intensively non-linear system, 50 stars, of all different sizes, all interacting with each other gravitationally. And yet it exhibits a dynamic stability (or dynamic equilibrium) where even though everything is moving, the overall shape of the galaxy remains more or less the same. It is analogous to a fountain; water continuously changing, yet the shape remaining overall the same. Both are complex systems.

9. Experimental Record Nine - A Complex System

What kind of attractor is this system in?

To see what else is possible, Go To Setup: Initial Velocity and set it from the default of 7 to a value of 30. Describe the change in behavior between between the default Initial Velocity, and an Initial Velocity of 30. What kind of attractor is present now?

10. Run Ten - Two Star Behavior

- Go to File: Open: 3.ini
- Go to Setup: Number of Stars = 2 stars; Initial Velocity = 0. Run and let the simulation go for a while.
  - Observe: the edges of the screen are "hard" and if a star hits one it bounces off like a billiard ball, sending it careening wildly. In time, however, the system should settle down again with the two stars orbiting each other.
  - Do Init and Run several times until you get a feel for what is happening.
  - The simulation can be very fast so it may be difficult to see what is happening but do the best you can.

10. Experimental Record Ten - Two Star Behavior

Describe the general behavior of the two stars across all your runs. Make a sketch of typical behavior. What kind of attractor is this?

11. Run Eleven - Two Stars of Different Initial Velocities

- SETUP = 2 stars;
- But now we want you to systematically change the INITIAL VELOCITY beginning at zero and advancing to 1, 2, 3, 4, etc. RUN and let each simulation go for a while.
  - The INITIAL VELOCITY is the velocity of stars at start of simulation (0 to 99) (clockwise motion around center of window).

11. Experimental Record Eleven - Two Stars of Different Initial Velocities

Now, go to **SETUP: INITIAL VELOCITY** and increase the initial velocity to 1, run and compare behavior with the previous. Continue increasing the velocity by 1's. At what initial velocity do you begin to see changes in the stars' behavior? What happens if you keep pushing the initial velocity higher and higher?
Describe how the behavior of the stars is changing.
We could think of the stars' initial velocity as analogous to "r" in the x-next equation. Write an explanation relating initial velocity to "r."

12. Run Twelve - The Three Body Problem

The three body problem is one that exists at the heart of chaos - sensitive dependence. The problem goes all the way back to Newton's day, implying that these people experienced the phenomena of chaos even if they did not know what to do with it. The equations Newton wrote to describe the behavior of objects in a gravitational field are very straight forward, deterministic equations, and can be easily solved for two bodies (planets, stars). Unfortunately, when you add a third body (planet, star) the system becomes so destabilized it never settles into a stable pattern. Which comes down to, when a system is running at high r value can we predict its future behavior. And the answer is "No!" That is, the future is permanently unknown, and unknowable

‘ FILE: OPEN = 3.ini; INITIAL VELOCITY = 0. Do INIT and RUN several times until you get a feel for what is happening.

12. EXPERIMENTAL RECORD TWELVE - THE THREE BODY PROBLEM

Describe the behavior of the three stars. Make a sketch, or sketches, of the behavior.

In terms of your observations in this experiment, in your own words, describe the three body problem in terms of ***attractors*** and ***r values***.

Think about this. Have you ever been in a relationship triangle? How close did the behavior of the three of you match that of the three body problem as shown by galaxy?

13. RUN THIRTEEN - THE COMPLEXITY OF COMPLEX BEHAVIOR

In the real world systems do not stay in the same state all the time, but evolve from state to state as conditions change. You can observe this in your own states; sometimes your are hyper, full of energy, bouncing off the walls, and at other times you are relaxed and serene. With this run we explore a system that changes behavior with time.

‘ FILE: OPEN = ***200.ini***. Do INIT and RUN several time. Observe long enough to know the complete range of behavior.

13. Experimental Record Thirteen - The Complexity of Complex Behavior

The behavior of this system changes both in r value and in the attractor state from the initial conditions. In the space below sketch across the page the various states the system passes through, and indicate their r values relative to each other, and their changing relative attractor states.

Sketch

Relative r value

Attractor state

What explanation can you offer for why this system is changing its behavior with time?

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Homework Assignment

Stretching the Imagination About Attractors

14. Thought Experiments About Attractors

OK, we have been looking at a lot of mathematical attractors, and by now you should have a good sense of the different kinds of attractors and their behavior. But from our point of view, this knowledge is useless if you cannot apply it to the real world, at least by analogy.

So, think about the following ideas and see what you can come up with. We will have a class seminar to discuss them. It is ok to collaborate with classmates in this project. And, you might be a lot more creative than we are, but for some of these we just cannot think of an example.

PHYSICAL/CHEMICAL WORLD EXAMPLES: For each of the attractors below, come up with as many physical world examples as you can think of.

Point

Circular

Outward Spiral

Limit Cycle (Periodic)

Strange (Chaotic)

BIOLOGICAL WORLD EXAMPLES: Come up with as many biological world examples as you can think of.

Point

Circular

Outward Spiral

Limit Cycle (Periodic)

Strange (Chaotic)

HUMAN HISTORY EXAMPLES: For each of the attractors below, come up with as many human history examples as you can think of. Human history includes any behavior associated with human activity, social, political, economic, etc. **Point**
Circular
Outward Spiral
Limit Cycle (Periodic)
Strange (Chaotic)

PERSONAL EXAMPLES: For each of the attractors below, come up with as many personal examples as you can think of. These are examples that apply to your personal life, either your mental worlds (dreams, fantasies, plans, etc), or your behavioral world. Note that you do not have to reveal any of these to anyone if you do not want to, so go ahead and take the risk of thinking about it. **Point**
Circular
Outward Spiral
Limit Cycle (Periodic)
Strange (Chaotic)

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