

Laboratory Instruction-Record Pages

LABORATORY EXPERIMENTS WITH THE X-next Bifurcation Diagrams

USING THE MODEL OF BRITISH BIOLOGIST ROBERT MAY, AND M.J. FEIGENBAUM
PROGRAMMED BY WILL FRANGOS

Geology 200 - Evolutionary Systems

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PURPOSE

We have explored the X_{next} system, but the X_{next} program only allows us to examine one r value at a time. Yet from the last experiment we know we want to compare changes in behavior from r value to r value, all at once, from $r = 1.0$ to $r = 4.0$. This program allows us to compare the behavior of X_{next} over a range of r values simultaneously, creating the *bifurcation diagram*.

Notes and Comments

- (When it comes time to do the formal experiments *follow the instructions below*. They are designed to systematically lead you through a series of observations.
- (You may consult with your instructor about questions that come up in your experiments.
- (You may discuss your experiments with other class members, but the results in your "Record of Experimental Results" must follow logically from your own observations and specific experimental results.

OPENING THE BIFURCATION AND GRAPHER PROGRAMS

These experiments require two programs, **Xnextbif.exe** to calculate the data, and a graphing program to plot the data (we use **Grapher**, but other graphing programs work). **Xnextbif.exe** has an icon under the Alife folder, while **Grapher** has an icon on the desktop; just double click and they open. Once you know how to run the programs they are easy, but getting started may be a hassle. We will lead you through the first experiment to reduce the hassles. Below, however, are two boxes, one for the procedures with **Xnextbif.exe** and the other for **Grapher**.

- ' **Xnextbif.exe** is available in the Geology Department computer lab, but if you want a copy for yourself just ask.
- ' **Grapher** is a commercial program, and unless you purchase a copy, is available only in Geology computer lab.

Both the Instructions for the experiments and the spaces for recording your experimental results are contained here.

EXPLORING THE X-NEXT BIFURCATION PROGRAM

XNEXTBif.EXE

' **X-nextbif** runs in a DOS window in Windows; screen capture below right. Click on desktop icon to load the program.

' You must create a directory to store your data files. You can save them to disc on the A:/ drive if they are not too big. Alternatively, you can create a folder on the S (students) drive to store your data files.

< Go to START: PROGRAMS: EXPLORER. Select the S drive and find the folder Xnextbif. Click on that folder and go to FILE: NEW: FOLDER and create a new folder naming it with your name, initials, or some other name you want.

' You are prompted to enter the parameters of the calculation.

└ Rmin/Rmax - range of r values in the run; r values > 4.0 cause the system to crash

└ Increment for r - use $.001$ as a default; this provides enough detail to see the fine structure

└ Initial value of X - use default of 0.2 unless directed otherwise

└ X 's per run - use 2000 ; this is enough to give most values of r time to attenuate about as much as they are going to attenuate.

└ X 's to plot - this is the memory hog in the program; we say here how many points to plot for each 0.001 increment of r . If we plot the last 200 iterations, at low values of r these all plot on top of each other because the system quickly attenuates to a single value and just keeps repeating it, plotting the same point over and over. At higher r values, where attenuation does not occur the 200 points (values of X) scatter vertically. The key is to find a value of X 's per run high enough to show the behavior of the system, but not so high to make the files too large to be managed. Plotting $100 X$'s gives a good ghost image of the bifurcation diagram; 200 shows some of the details.

└ Name of output file - name the file using some convenient name. You must specify the drive, for example

S:\xnextbif\lsf\bif1.dat

Be sure to add the .dat extension. Once Xnextbif.exe is done it prints the output to this folder/file.

```

XnextBif
Auto
Calculate X-Next Bifurcation Diagram
as fctn of Rmin & Rmax --wf, 3iv01 ver 1.4

Enter Rmin Rmax: 2.5 4.0
Enter Increment for R: .001
Enter initial X value: .2
Enter # of X's per run: 5000
Enter # of X's to plot (last N of the run): 500

Name of the OUTPUT file : bif6.dat_
  
```

These data files tend to be very large and can take many minutes to 10's of minutes to print so be forewarned. The experiments below should take only a few minutes to plot

EXPLORING GRAPHER

Grapher is a commercial Windows based program, and is available on all the computers in the lab, but you can use any program that allows you to build diagrams from raw data, such as Excel or Quattro Pro.

L As with most programs that have a lot of power and options Grapher is complex to use, but once you know what you want to do, the procedures are simple.

L Find the Grapher icon on the desktop and double click it to open the program. It should open with a new page. Go to VIEW:PAGE to enlarge and center it if it is not already.

L Load the data file: go to GRAPH: NEW GRAPH: LINE/SYMBOL and under "Look in" find your data file and double click it. Grapher will load the data and open the window to the right.

L In the GRAPH 1 - LINE/SYMBOL PLOT 1 window:

< Go to LINE-FILL: LINE PROPERTIES and set the Style to INVISIBLE (first choice below the first line).

< Back to LINE-PLOT: go to SYMBOL (lower right), click on cross hair, select the black dot and set the size to .01 inch. Also set frequency to 1.

< Click OK and Grapher graphs your data into a bifurcation diagram.

< If you want to change anything about the graph, select it by clicking on it and then right click to PROPERTIES.

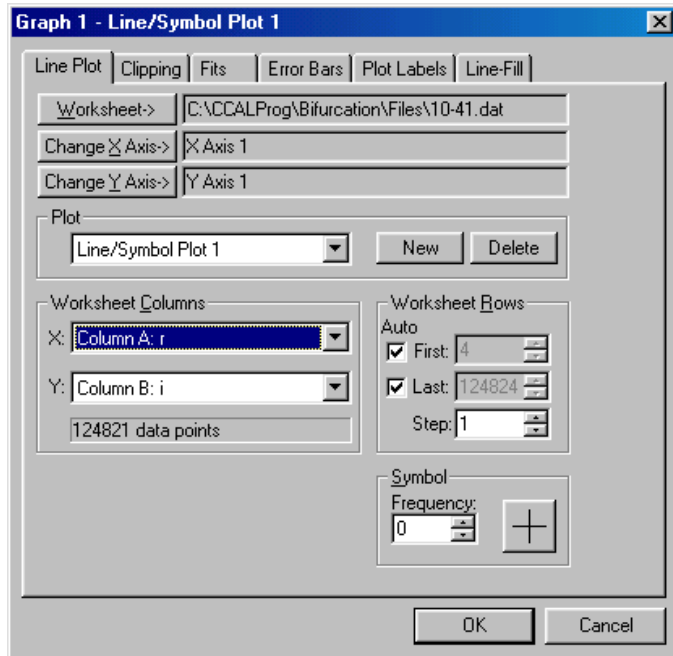
< In the left side tool bar click on the letter A, then click it over the page and a text window opens. Type the following:

1. Your name
2. Increments =
3. Initial X =
4. X's per run =
5. X's to plot =

< Click OK and that text shows up on the diagram. You may move it anywhere you want.

< OR ALTERNATIVELY - Use WINDOWS EXPLORER to find the data file in its directory. Double click the file and it will open in NotePad. COPY the experimental parameters at the top and paste them into Gapher.

L Go To FILE: PRINT (wait, . . . wait, . . . wait until it prints :-)



Experiment One - Bifurcation Diagrams

The Basic Bifurcation Diagram

1. RUN ONE - COMPLETE BIFURCATION DIAGRAM (in lab we talk you through this first example)

- L Open XnextBif and fill in the following parameters
 - < Enter Rmin Rmax **1.0 4.0**
 - < Enter Increment of r **.001**
 - < Enter Initial X value **.2**
 - < Enter # of X's per run **2000**
 - < Enter # of X's to run (last N of run): **25**
 - < Name of Output File **S:\XnextBif\???\?????.dat**
 - < Output to Screen **No**
- L Load data file into Grapher following the instructions above.
- L FILE: PRINT

1. EXPERIMENTAL RECORD ONE - COMPLETE BIFURCATION DIAGRAM

Explain in writing exactly what this diagram represents (in terms your younger sibling would understand; if you don't have a younger sibling, borrow one)

Label the 1st, 2nd, and 3rd bifurcations on the diagram. Can you discriminate any higher bifurcations?

Do bifurcations come *faster* or *slower* with increasing r ? (*The Feigenbaum number is the ratio between increasing r and increasing rates of bifurcation; the same number turns up in various chaotic systems, it is about 4.669, and it is a fractal relationship.*)

Bracket and label on the bifurcation diagram the zone you think represents the *complex realm*.

Bracket and label on the bifurcation diagram the zone you think represents *deterministic chaos*.

Study the bifurcation diagram. Identify at least three interesting or unusual things about, or in, the diagram that indicate interesting and/or unexpected behavior of the system.

- 1.
- 2.
- 3.

Draw boxes around two parts of the diagram where you would like to see more detail.

Experiment Two - Bifurcation Diagrams Honing in on the Complexity Realm

2. RUN TWO - HONING IN ON THE COMPLEXITY REALM

L Create and print a bifurcation diagram with the following parameters

- < Enter Rmin Rmax **3.45 3.7**
- < Enter Increment of r **.001**
- < Enter Initial X value **.2**
- < Enter # of X's per run **2000**
- < Enter # of X's to run (last N of run): **200**
- < Name of Output File **S:\XnextBif\???\?????.dat**
- < Output to Screen **No**

2. EXPERIMENTAL RECORD TWO - HONING IN ON THE COMPLEXITY REALM

Return to the Diagram for Experiment One and draw two vertical lines through it showing the range of r this second diagram represents.

Describe any new features that appear in this more detailed diagram that are not clearly visible in the first diagram.

Experiment Three - Bifurcation Diagrams Bifurcation Windows in the Chaos Realm

The last diagram narrowed the r range down to 3.45 to 3.7. Let's narrow it down even more, to 3.6 - 3.65.

3. RUN THREE - BIFURCATION WINDOWS IN THE CHAOS REALM

L Create and print a bifurcation diagram with the following parameters

- < Enter Rmin Rmax **3.60 3.65**
- < Enter Increment of r **.0001**
- < Enter Initial X value **.2**
- < Enter # of X 's per run **2000**
- < Enter # of X 's to run (last N of run): **100**
- < Name of Output File **S:\XnextBif\???\?????.dat**
- < Output to Screen **No**

3. EXPERIMENTAL RECORD THREE - BIFURCATION WINDOWS IN THE CHAOS REALM

On the diagram for the Second experiment draw two vertical lines indicating the 3.6 to 3.65 range, just so you have an appreciation of the new scale of observation we are at.

Run Three - what new feature appears at this higher magnification that are not (so easily) seen in the previous? Circle them on the diagram and briefly sketch and describe them below.

4. RUN FOUR - CLIPPING THE BIFURCATION DIAGRAM

L We are going to zoom in one more time, in this case to just the upper arm of the diagram you just plotted; clipping it enlarges it in the final printing. Open the file in Grapher and save it under a different name. Click on the diagram to select it, then right click to PROPERTIES: CLIPPING. Set **Ymin** to 0.75, click ok, save and print.

At 100 plots per r this is not as detailed a diagram as if it were 200 plots per r , but it is enough to see what is going on.

4. EXPERIMENTAL RECORD FOUR - CLIPPING THE BIFURCATION DIAGRAM

Return to the first diagram of the full bifurcation diagram and draw a box around the area this new, clipped diagram represents. Estimate the percent of the detail in the clipped diagram that is *not* visible in the full bifurcation diagram.

% of detail not visible in full diagram

How much longer do you think we could continue enlargements and still see new detail?

Find and write below some values of r from Experimental Record Four where you think new details would emerge.

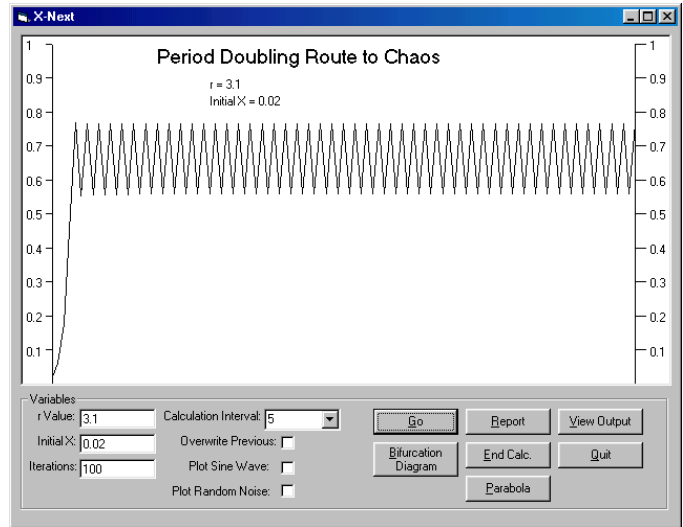
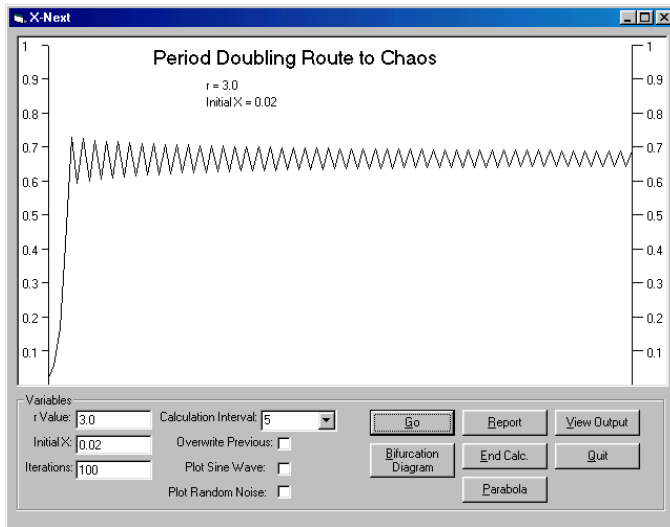
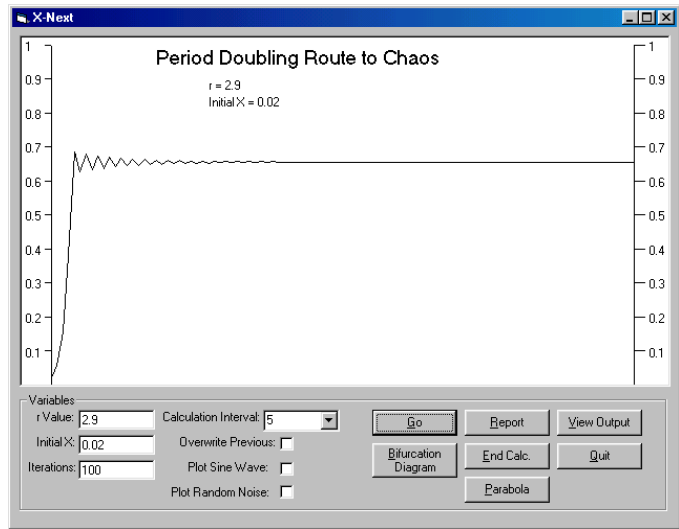
Experiment Four - Bifurcation Diagrams Instability and Bifurcations

Recall the X_{next} experiment. At $r = 2.9$ the system attenuates quickly (screen capture to right). At $r = 3.1$ the system oscillates with precision out to all decimal places (screen captures below). The first system is clearly a point attractor, and the latter an oscillatory attractor. But, what happens in between? At $r = 3.0$ the system never completely stabilizes to a point or an oscillatory attractor.

The implications of this are that as a system approaches a bifurcation it goes from relatively regular behavior, to behavior that becomes unstable, and back to behavior that is regular again.

But is that really what happens, or are we just jumping to a wrong conclusion from limited data, from only r 's of 2.9, 3.0, and 3.1?

More specifically, does the instability occur only



as the system approaches a bifurcation, and then quickly stabilize immediately after the bifurcation takes place, or does the zone of instability exist both before and after the bifurcation, or does it do something else entirely?

We want you to do two things here. First, predict what you think the behavior will be, based on what you have learned about this system. Second, devise and conduct an experiment to test your prediction.

5. EXPERIMENTAL RECORD FIVE - PREDICTION OF BEHAVIOR WHILE ENTERING AND LEAVING A BIFURCATION

Thought Experiment: Think about the situation describe above. Predict and write below a hypothesis of what you think the behavior of the system will be as it enters and passes through the first bifurcation. Or, sketch how the system will look in a bifurcation diagram.

In this next bifurcation diagram we change the strategy from that used in the previous calculations. Previously, we wanted the system to attenuate as well as it was able in 2000 calculations, and then we plotted the last 50 or 100 iterations to observe its final behavior.

In this experiment it is the early behavior we are interested in, before it has attenuated, especially for low r values. This will tell us how quickly a system attenuates; that is, whether it stabilizes quickly to either a point or oscillatory attractor, or whether it is unstable and takes a long time to settle in.

And since we start with an initial $r = .2$, which is relatively low, we want to filter out those first few calculations where the value of X is rising toward it attractor state.

Run Six - Instability and Bifurcations

L Create and print a bifurcation diagram with the following parameters

- < Enter Rmin Rmax **2.40 3.6** (*this captures the 1st and 2nd bifurcations*)
- < Enter Increment of r **.01** (*calculating fewer r 's spreads the data points allowing us to see them better*)
- < Enter Initial X value **.2**
- < Enter # of X 's per run **25** (*if it attenuates quickly it will happen in these first few iterations; if not we should see a vertical spread of X values*)
- < Enter # of X 's to run (last N of run): **20**
- < Name of Output File **S:\XnextBif\???\?????.dat**
- < Output to Screen **No**

6. EXPERIMENTAL RECORD SIX - INSTABILITY AND BIFURCATION

Observations: To understand what is going on, begin by making the following observations.

1. Examine the diagram to get a sense of the scale on the y axis along which values of X are plotted.

Grapher has plotted every 0.1 tick mark. Sketch in some tick marks that represent two decimal places of X, e.g. 0.51, 0.52 etc. Then visualize the scale of three decimal places.

Notice that data points that differ by at least 0.01 X plot as distinct points vertically (because you set the point diameter at 0.01; observe diagram), and we spread the values of r out enough that points plot distinctly horizontally on the diagram (observe diagram).

2. Draw a vertical line up from $r = 2.8$ through the diagram.

Can you see how the system oscillates between two values while attenuating?

3. Go to START: PROGRAMS: EXPLORER. Go to the .dat file you created for this experiment; double click and it will open in WordPad. Observe the three columns of data, the first is the r value, the second the X value, and the third the iteration. In the sample to the right we copied the last 5 iterations of $r = 2.79$ (20-25), the last 20 iterations of $r = 2.8$, and the fifth, and first plotted, iteration of $r = 2.81$.

4. Compare back and forth between the X values for $r = 2.8$ in the table with the diagram you printed until you can see how the individual X values have plotted in the diagram.

5. Go to the data file in WordPad and observe that for $r = 3.2$ the X values oscillate between two numbers out to 5 decimal places for all iterations from 6 - 25. Observe what that data looks like on the diagram.

6. If you want compare data in the WordPad file for other values of r with how they appear in the diagram.

<u>r value</u>	<u>X value</u>	<u>Iter</u>
2.7900	0.64228	20
2.7900	0.64102	21
2.7900	0.64201	22
2.7900	0.64123	23
2.7900	0.64185	24
2.7900	0.64136	25
2.8000	0.61520	5
2.8000	0.66284	6
2.8000	0.62575	7
2.8000	0.65572	8
2.8000	0.63210	9
2.8000	0.65114	10
2.8000	0.63604	11
2.8000	0.64818	12
2.8000	0.63852	13
2.8000	0.64627	14
2.8000	0.64009	15
2.8000	0.64505	16
2.8000	0.64109	17
2.8000	0.64426	18
2.8000	0.64173	19
2.8000	0.64376	20
2.8000	0.64214	21
2.8000	0.64343	22
2.8000	0.64240	23
2.8000	0.64323	24
2.8000	0.64256	25
2.8100	0.61446	5

Writing Assignment: Examine the bifurcation diagram as it enters and passes through the first and second bifurcations. Describe below how the stability of the system changes as it passes through a bifurcation.

Experiment Six - Bifurcation Diagrams

Chaos vs Deterministic Chaos

The common definition of chaos is *utter disorder and confusion*, but as we have explained that is not what we are talking about here, or what James Gleick meant in his book titled “Chaos” the first and still one of the best discussions of chaos theory. The question is, “how can you tell utter disorder and confusion apart from *deterministic chaos*, which is what chaos theory deals with?”

Make an Observation: For example, examine again your bifurcation diagrams. There are regions in the diagrams where pattern and order exists, such as the curving lines of the point attractors, and the zones of bifurcation. But there are other regions where no discernable pattern shows up, the regions of chaos. Study some of these regions of chaos on your bifurcation diagrams and see if you can detect any discernable patterns to the data.

Looking at these chaos zones within a bifurcation diagram how can we determine whether we are observing old fashioned chaos or deterministic chaos? Sometimes the art of analysis is finding just the right way of transforming the data into a form that brings out its patterns and meaning. Here there is a neat mathematical conversion to tell us when we are dealing with chaos and when deterministic chaos.

RUN SEVEN - CHAOS VS DETERMINISTIC CHAOS

Open the X_{next} program.

- L Set $r = 4$ for 500 iterations and click GO. Do you observe any pattern in the graph? Try 3.9999, and 4.00001.
- L Now select OVERWRITE PREVIOUS and PLOT RANDOM NOISE (X values created by a random number generator). Click GO several times; observe and compare the graphs; do any show a pattern?

7. EXPERIMENTAL RECORD SEVEN - CHAOS VS DETERMINISTIC CHAOS

One visible difference does commonly, but not always, occur between plots at high r (3.90000 to a little over 4.0) and the PLOT RANDOM NOISE. Can you detect it? Describe or draw what you observe.

Now, select PLOT RANDOM NOISE and then X-Y PLOT. Try it at 5000 iterations (set calculation interval to zero). Describe the resulting plot (sketch what it looks like).

Now, set r to 4.0, **uncheck** PLOT RANDOM NOISE, leave iterations at 5000 and click X-Y PLOT. Describe how this plot differs from the previous (sketch what it looks like).

Do the X-Y PLOT for the following values of r and sketch the results.

r = 2.5	r = 2.7	r = 2.9	r = 3.1
r = 3.3	r = 3.5	r = 3.7	r = 3.

The X, Y graphs are generated from the string of numbers generated either by the random number generator in the computer, or by the iteration of X_{next} . The algorithm is

- L Make the first number in the string X, and the second number Y, and plot the point.
- L Then make the second number in the string X, and the third number Y, and plot the point.
- L Continue to the end of the string of numbers.

Or, in other words the X_{next} in the logistic equation $X_{next} = rX(1 - X)$ becomes Y, as in $Y = rX(1 - X)$.

In a string of random numbers each new number is unconnected with the previous, so the second number has no relationship to the first, and the third number has no relationship to the second, and likewise through the entire string of random numbers. As a result the points plot as a random scatter.

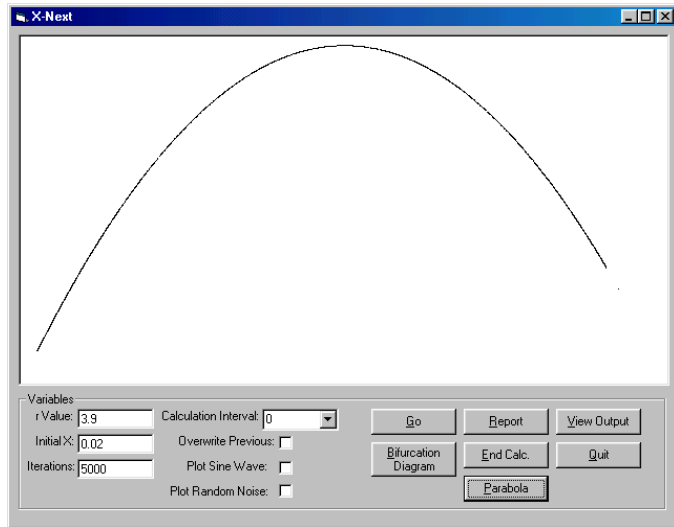
With X_{next} the second number is dependent on the first, and the third on the second (which is dependent on the first) and likewise through the entire string of numbers. What makes deterministic chaos **deterministic** is that the patterns and outcomes are recursive. That is, every calculation of a chaos system is dependent on and the result of all the previous outcomes. In other situations we refer to this as **contingency**, the future is contingent, dependent on, on the past.

The reason that deterministic chaos systems plot as a parabola on an X, Y graph is that, with a little manipulation the X_{next} equation can be shown to be a quadratic equation ($Y = aX^2 + bX + C$), and a quadratic equation is the equation for a parabola. If you want more about this see the last page.

8. EXPERIMENTAL RECORD EIGHT - THE PARABOLA PLOTS AT DIFFERENT R VALUES

The last experiment you did was to sketch the parabolic outcomes for various values of r . Review those sketches and observe that at low r values the parabolas are very incomplete, but the higher the r the more complete they become, until at $r = 4.0$ the parabola is complete. Notice, however, that even at $r = 3.9$ (screen capture to right) the parabola is still not complete on the right side.

Provide an explanation for the way in which the parabola fills out with increasing values of r . (Hint: X on the horizontal axis runs from 0 to 1, and Y on the vertical axis runs from 0 to 1).



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